

**32.20** A 20ft length of uninsulated pipe with 2in O.D. has a surface temperature of 180°F. The ambient air is 70°F. What is the heat loss by natural convection?

- A. 200  $\frac{Btu}{hr}$
- B. 700  $\frac{Btu}{hr}$
- C. 1500  $\frac{Btu}{hr}$
- D. 2100  $\frac{Btu}{hr}$

Convection problems follow **Newton's Law of Cooling**.

$$\dot{Q} = hA\Delta T$$

Determine the surface area for the cylindrical pipe.

$$A = \pi DL = \pi \left( \frac{2}{12} ft \right) (20ft) = 10.5ft^2$$

Find  $\Delta T$ .

$$\Delta T = 180^\circ F - 70^\circ F = 110^\circ F$$

Determining the convection heat transfer coefficient when it is not given takes a fair amount of effort and several assumptions. Start by referring to the section on **Natural (Free) Convection**, specifically the **Long Horizontal Cylinder in Large Body of Stationary Fluid**. Write the formula for the average convection coefficient.

$$\bar{h} = C \left( \frac{k}{D} \right) Ra_D^n$$

The Raleigh number is found with the following formula and used in conjunction with the table to determine the values of the constants  $C$  and  $n$ .

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3Pr}{\nu^2}$$

All parameters should be looked up at the **Film Temperature** of the air, which is simply the average of the surface temperature and the ambient temperature.

$$T_f = \frac{T_s + T_\infty}{2} = \frac{180^\circ F + 70^\circ F}{2} = 125^\circ F$$

Search for **Properties of Air** and find two different tables which provide thermal conductivity, Prandtl number, absolute viscosity, and density. Calculate the kinematic viscosity.

$$k \approx 0.016 \frac{Btu}{hr \cdot ft \cdot ^\circ F}$$

$$Pr \approx 0.71$$

$$\mu \approx 130 \times 10^{-7} \frac{lb_m}{s \cdot ft}$$

$$\rho \approx 0.075 \frac{lb_m}{ft^3}$$

$$\nu = \frac{\mu}{\rho} = \frac{130 \times 10^{-7} \frac{lb_m}{s \cdot ft}}{0.075 \frac{lb_m}{ft^3}} = 1.73 \times 10^{-4} \frac{ft^2}{s}$$

Search **Coefficient of Thermal Expansion** and use the formula intended for an ideal gas, which is an adequate approximation. Use absolute i.e. Rankine temperatures.

$$\beta = \frac{2}{T_s + T_\infty} = \frac{2}{640R + 530R} = 0.0017R^{-1}$$

Calculate  $Ra$ . Since the temperature term is a difference, it's magnitude would be the same whether Fahrenheit or Rankine was used, therefore it is acceptable to cancel the temperature units out.

$$Ra_D = \frac{\left(32.2 \frac{ft}{s^2}\right) (0.0017R^{-1}) (180^\circ F - 70^\circ F) \left(\frac{2}{12} ft\right)^3 (0.71)}{\left(1.73 \times 10^{-4} \frac{ft^2}{s}\right)^2} = 661,317$$

Referring to the table, obtain values for the constants  $C$  and  $n$ .

$$C = 0.48$$

$$n = 0.25$$

Calculate  $\bar{h}$ .

$$\bar{h} = (0.48) \left( \frac{0.016 \frac{Btu}{hr \cdot ft \cdot ^\circ F}}{\frac{2}{12} ft} \right) (661,317)^{0.25} = 1.31 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}$$

Determine the heat transfer.

$$\dot{Q} = hA\Delta T$$

$$\dot{Q} = \left(1.31 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}\right) (10.5 ft^2) (110^\circ F) = 1518 \frac{Btu}{hr}$$

**Answer C**